AN APPROXIMATE GALERKIN METHOD IN QUASILINEAR EQUATIONS OF MATHEMATICAL PHYSICS

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The solution of the quasilinear heat-conduction equation is considered by averaging the variable coefficient and by the Bubnov–Galerkin method.

Many problems of mathematical physics, related to heat conduction, filtration, diffusion, and other processes of heat and mass transfer, reduce to solution of nonlinear equations of parabolic type, written conveniently in the following form [1, 2]:

$$\frac{d}{d\eta}\left[\varkappa(U)\frac{dU}{d\eta}\right]=-2\eta\frac{dU}{d\eta},\quad \eta=\frac{x}{2\sqrt{t}}, \tag{1}$$

where $\varkappa$ is the coefficient of piezoconduction (heat conduction, diffusion, etc.).

A solution of (1) is sought with boundary conditions

$$U(0)=1,\quad U(\infty)=0. \tag{2}$$

For the linear dependence $\varkappa(U) = 1 + \alpha U$, we obtain

$$\frac{d}{d\eta}\left[\varkappa\frac{d\varkappa}{d\eta}\right]=-2\eta\frac{d\varkappa}{d\eta}. \tag{3}$$

The weak dependence of the solution of Eq. (1) on the variable coefficient $\varkappa(U)$ makes it possible to determine its value by approximate methods [3, 4]. Taking this into account, we put

$$\varkappa=\varkappa_0-(\varkappa_0-\varkappa_h)\left(1-\frac{x}{l}\right)^n,\quad 2\leqslant n\leqslant 3,\quad l(t)=\beta\sqrt{t}, \tag{4}$$

where $l(t)$ is the penetration depth of the perturbation; β, a constant coefficient; and $\varkappa_h$ and $\varkappa_0$, are values of the unknown coefficient at the perturbation front.

We define the average value

$$\tilde{\varkappa}=\frac{1}{l}\int_0^l \varkappa dx=\frac{n\varkappa_0+\varkappa_r}{n+1}. \tag{5}$$

An estimate of various methods of linearization of equations of type (1) was carried out in [4]. Application of the comparison theorem and of the L. S. Leibenzon linearization method showed that even for a twofold change of the coefficient mentioned the results differ by 15-20%. Most convenient is the method of averaging partial derivatives of potential functions with respect to time.

Calculations have shown that for boundary conditions of the first kind n = 2 gives high accuracy. Carrying out the averaging for boundary conditions of the second kind gives n = 3 [4]. Starting from this fact, we assume that in Eq. (4) the power varies within the limits $2 \leqslant n \leqslant 3$.

We provide calculations for n = 2, corresponding to boundary conditions of the first kind for the half plane:

$$\tilde{\varkappa}=(3+\alpha)/3. \tag{6}$$

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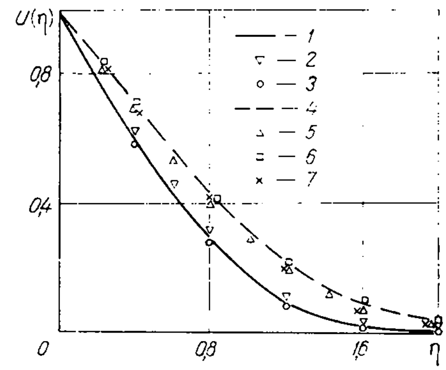

Fig. 1. Solutions of Eq. (1): 1) numerical solution for α = 0.1 [1]; 2) zeroth approximation according to Bubnov–Galerkin [5] for α = 0.1; 3) by Eq. (7) for α = 0.1; 4) numerical solution for α = 1 [1]; 5) first approximation according to Bubnov–Galerkin for α = 1 [5]; 6) according to Eq. (11); 7) graphical solution for α = 1.

The approximate solution of the problem under consideration is then

$$U = \operatorname{erfc}\left(\eta \sqrt{3/(3+\alpha)}\right). \tag{7}$$

Consequently, the solution of the nonlinear equation (1) can be represented as a solution of the corresponding linear equation with an argument correction

$$U = \operatorname{erfc}(b_i \eta), \quad i = 1, 2, \ldots, m. \tag{8}$$

We specify the coefficients b_i by the Bubnov–Galerkin method. As a zeroth approximation we choose

$$b_0 = \sqrt{3(3+\alpha)}. \tag{9}$$

The coefficient α for oil deposits is on the order of 10^{-2}. In the present calculations we take α = 0.1.

Results of the calculation are given in Fig. 1.

The calculations performed here show that averaging of a variable coefficient of quasilinear parabolic equations frees us from the necessity of performing awkward calculations in choosing trial functions by the Bubnov–Galerkin method. Refinement of the calculations by the Bubnov–Galerkin method gave the following values of the coefficient b_m:0.97 and 0.73 for α = 0.1 and 1, respectively. It follows from the statement of the problem that b_m = 1 for α = 0. Empirically this dependence is given by the equation

$$b = 1 - 0.27\alpha. \tag{10}$$

The final solution of the problem under consideration is, then, for α = 0-1,

$$U = \operatorname{erfc}[\eta(1 - 0.27\alpha)]. \tag{11}$$

The possibility of representing the solution of the nonlinear equation (1) in terms of solutions of linear equations in form (8) allows one to obtain simpler methods of solution.

We represent (1) in the form

$$a(\varphi)\frac{d^2\varphi}{d\eta^2} + 2\eta\frac{d\varphi}{d\eta} = 0, \tag{12}$$

where $\varphi(U) = \int_0^U (1 + \alpha U)\, dU$; $\alpha(\varphi) = \sqrt{1 + 2\alpha\varphi}$.

We decompose the interval of variation of the unknown function into n parts. Near the i-th point we put $a(\varphi_i)$ = const. The solution of (12) for the i-th point can then be represented in the form

$$\varphi_i = \operatorname{erfc}[\eta/\sqrt{a(\varphi_i)}]. \tag{13}$$

A similar linearization was earlier used [2] for the whole region of the solution of [12]. In the given case, by linearizing the equation in each point of the region we obtain a more accurate solution. Equation (13) is transcendental in φ_1, and its solution can be found easily, e.g., by graphical means. From the given graphical solution one can determine the correction coefficient b_0 for the zeroth approximation. In the given case for $\alpha = 1$ this coefficient equals 0.7. Further refinements need to be made by the above described Bubnov–Galerkin method.

NOTATION

U, unknown function; $\varkappa_0$ and $\varkappa_h$, given constant values of the coefficient $\varkappa$; α, a constant; erfc, probability integral function; x, distance coordinate; t, time; η, a variable dependent on x and t; b_i, correction coefficients.

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